

My FFE : Frequently Forgotten Equations

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Trigonometric Function

additional theorem of trigonometric function

$$\begin{aligned}\sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}\end{aligned}$$

formulas for half angles

$$\begin{aligned}\sin^2 \frac{\alpha}{2} &= \frac{1 - \cos \alpha}{2} \\ \cos^2 \frac{\alpha}{2} &= \frac{1 + \cos \alpha}{2} \\ \tan^2 \frac{\alpha}{2} &= \frac{1 - \cos \alpha}{1 + \cos \alpha}\end{aligned}$$

formulas for double angles

$$\begin{aligned}\sin 2\alpha &= 2 \sin \alpha \cos \alpha \\ \cos 2\alpha &= \cos^2 \alpha - \sin^2 \alpha \\ &= 2 \cos^2 \alpha - 1 \\ &= 1 - 2 \sin^2 \alpha \\ \tan 2\alpha &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha}\end{aligned}$$

formulas for triple angles

$$\begin{aligned}\sin 3\alpha &= 3 \sin \alpha - 4 \sin^3 \alpha \\ \cos 3\alpha &= 4 \cos^3 \alpha - 3 \cos \alpha \\ \tan 3\alpha &= \frac{3 \tan \alpha - \tan^3 \alpha}{1 - 3 \tan^2 \alpha}\end{aligned}$$

Euler's formura : θ is real

$$\begin{aligned} e^{i\theta} &= \cos \theta + i \sin \theta \\ \cos \theta &= \frac{1}{2}(e^{i\theta} + e^{-i\theta}) \\ \sin \theta &= \frac{1}{2i}(e^{i\theta} - e^{-i\theta}) \\ (\cos \theta + i \sin \theta)^m &= \cos m\theta + i \sin m\theta \quad (\text{de Moivre}) \end{aligned}$$