e
The meaning of the base of natural logarithm's definition

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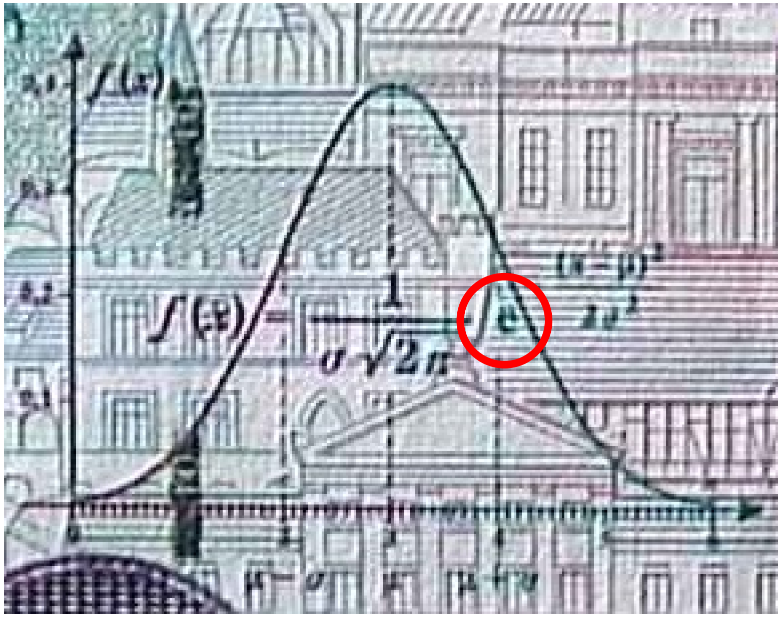
e is

- The most famous transcendental number (or π)
  - 2.7182818284590452353602874713526 62...
  - Symbol honors Euler
  - Mathematics, science, ...
  - Zehn Deutsche Mark
- Definition

$$e \equiv \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$

e is

- Everyone knows *e* (and π)
  - 2.718281828459045235360287471352662...
  - Symbol honors Euler
  - Mathematics, science, ...
  - Zehn Deutsche Mark →



- Definition

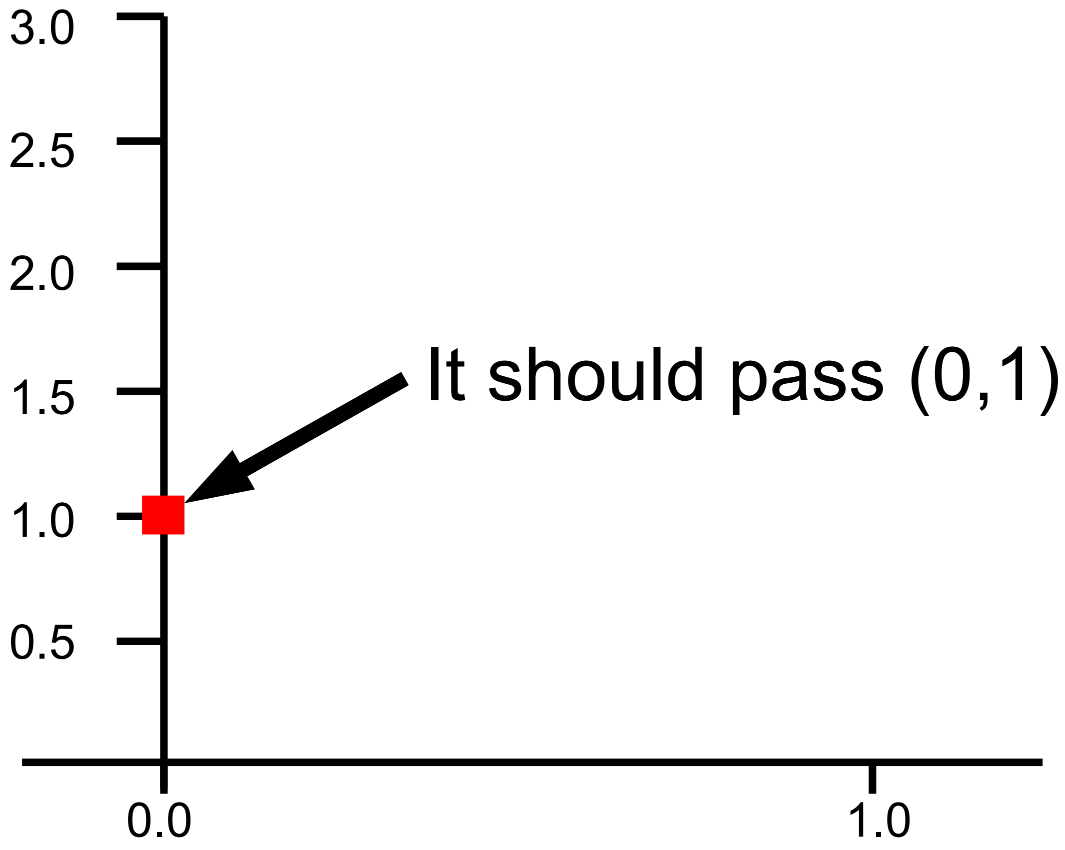
$$e \equiv \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}$$

- But why this definition?

Assumptions of *e*

- power to 0 of any number is 1. So is *e*.

$$e^0 = 1$$



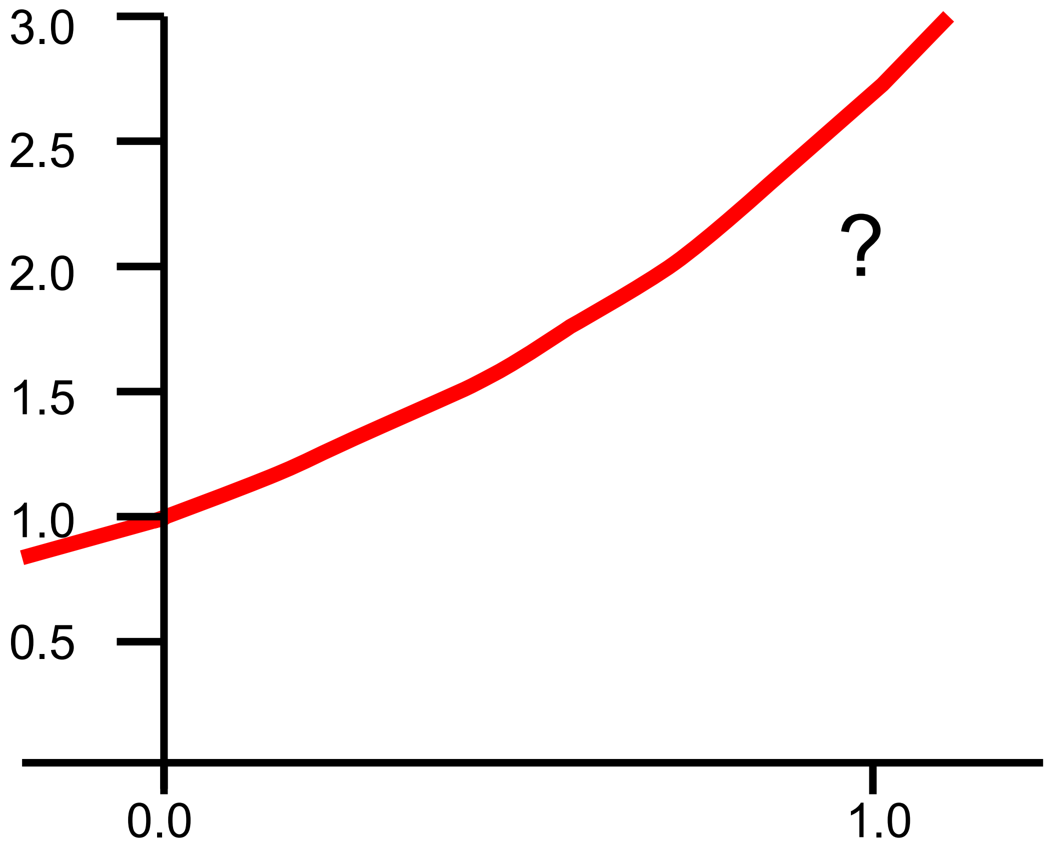
Assumptions of *e*

- power to 0 of any number is 1. So is *e*.

$$e^0 = 1$$

- derivative of *e*^*x* is *e*^*x*.

$$\frac{d}{dx}e^x = e^x$$



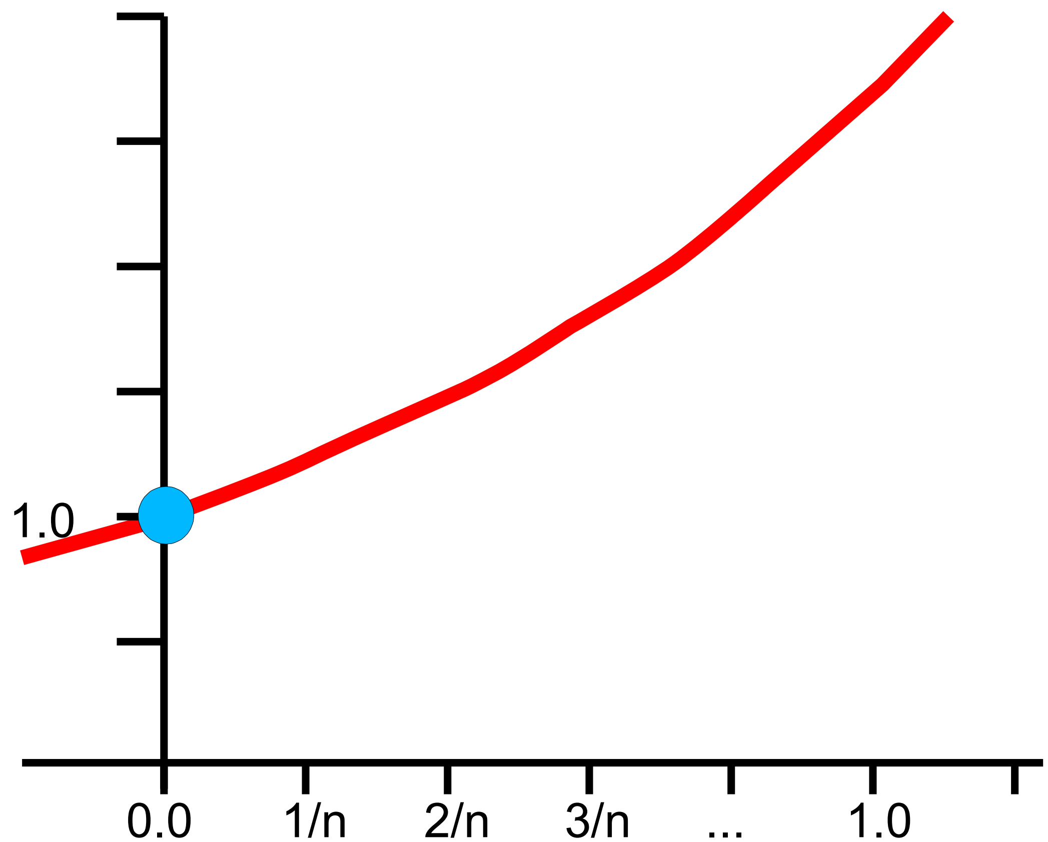
Approximation of *e*

- Assumption

$$
\begin{aligned}
e^0 &= 1 \\
\frac{d}{dx}e^x &= e^x
\end{aligned}
$$

- Subdivide 0.0 to 1.0 by n

$$y_0 = e^0 = 1$$

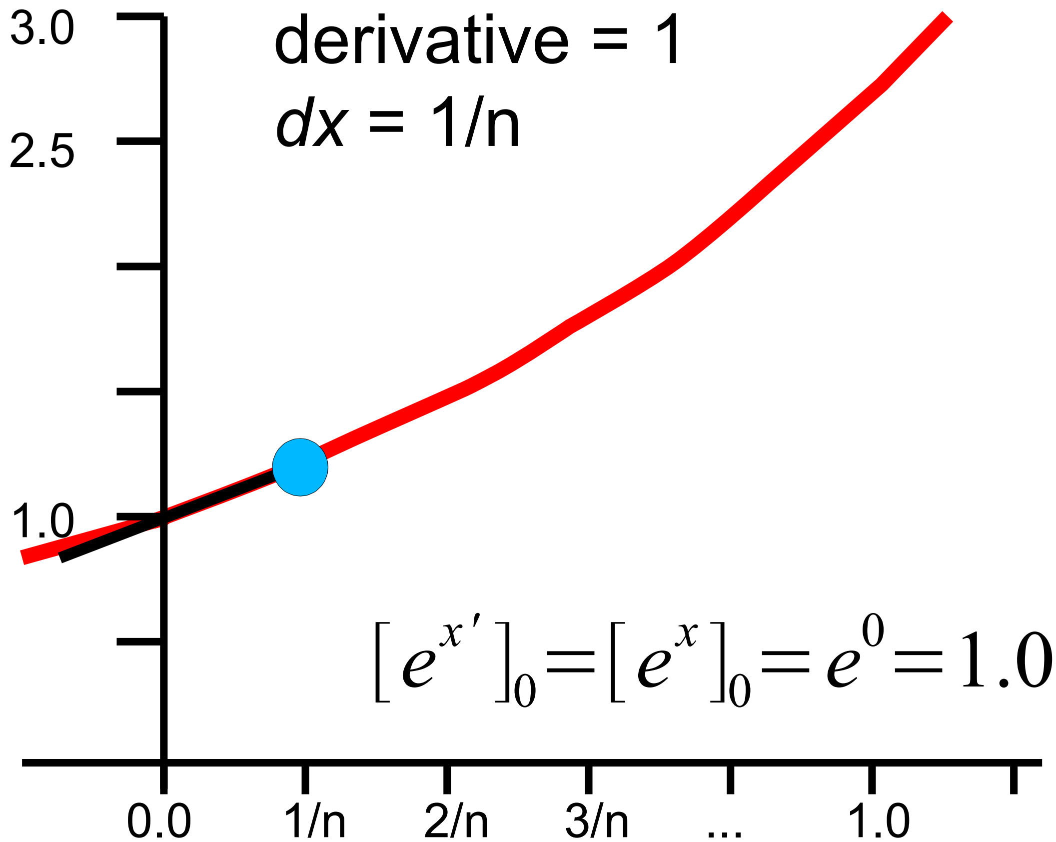


y1 = ?

- Approximate with Taylor expansion

$$= y_0 + \left.\frac{de^x}{dx}\right|_{x=0} \frac{1}{n}$$

$$= 1 + 1 \cdot \frac{1}{n}$$



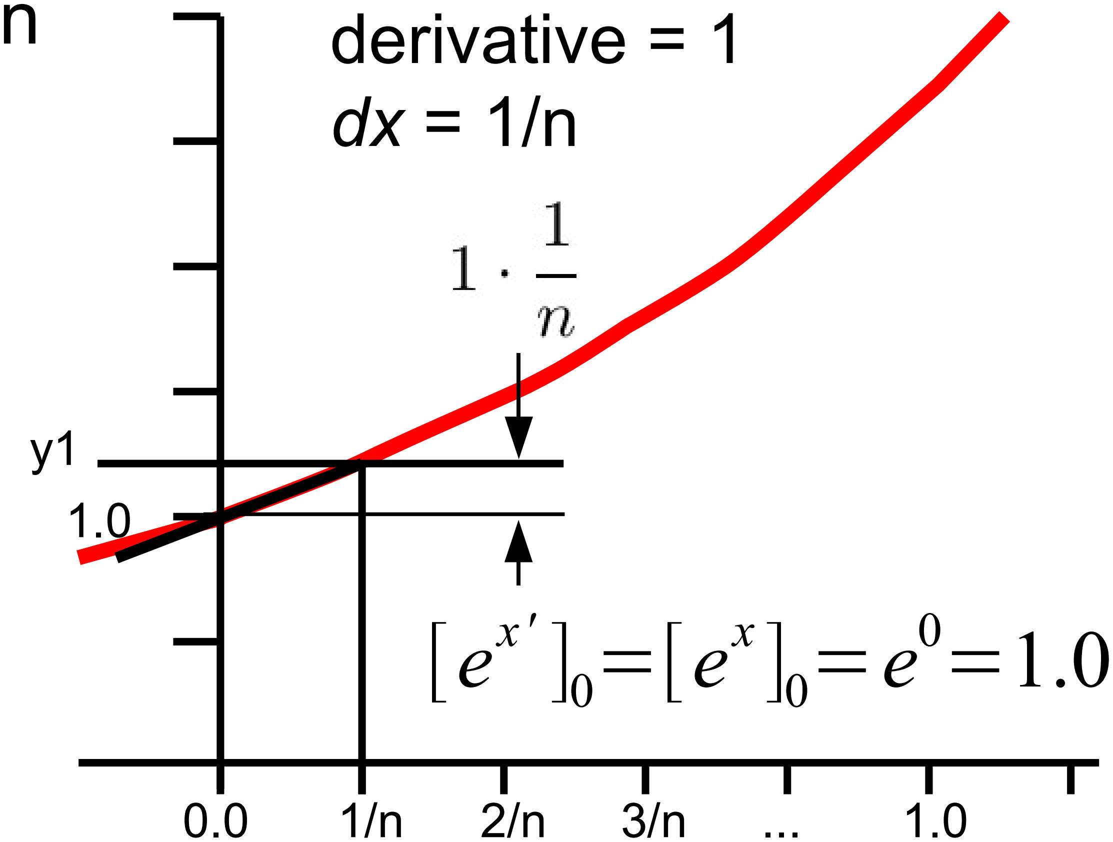
$$[e^{x'}]_0 = [e^x]_0 = e^0 = 1.0$$

y1 =

- Approximate with Taylor expansion

$$
\begin{aligned}
&= y_0 + \left.\frac{de^x}{dx}\right|_{x=0} \frac{1}{n} \\
&= 1 + 1 \cdot \frac{1}{n}
\end{aligned}
$$

$$y_1 = 1 + \frac{1}{n}$$



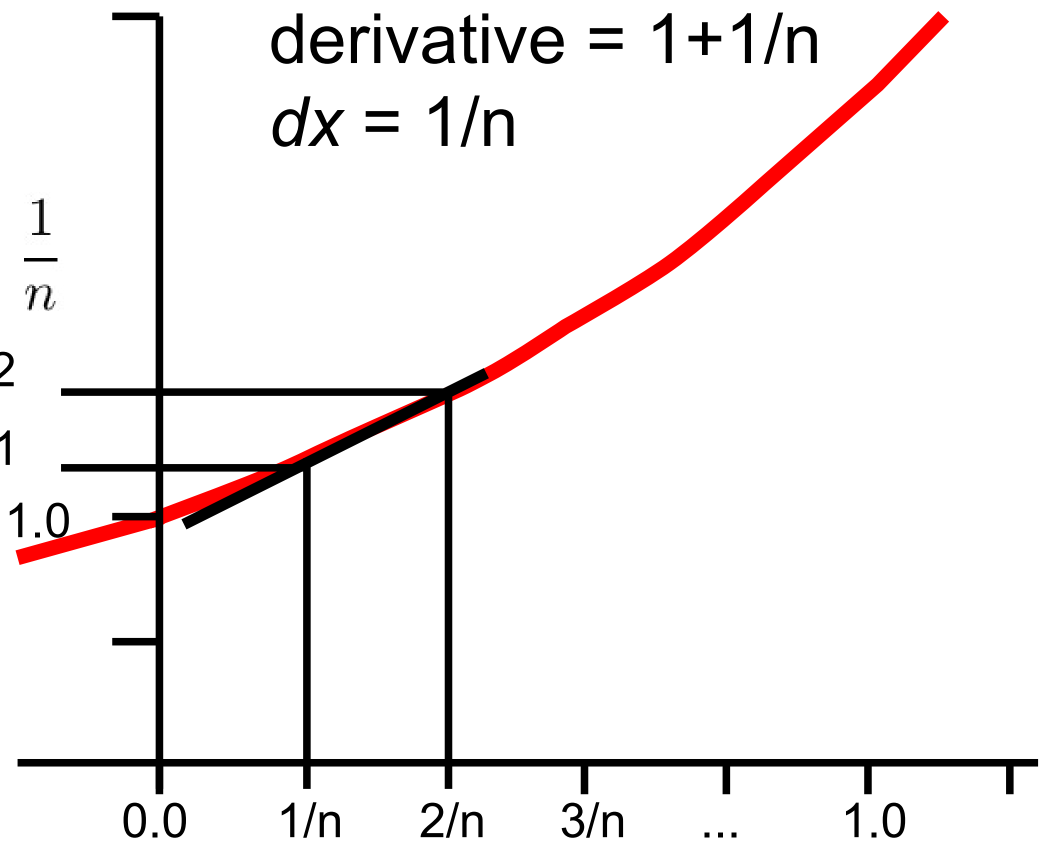
y2 =

- Taylor expansion again

$$= y_1 + \left(1 + \frac{1}{n}\right)\frac{1}{n}$$

$$= \left(1 + \frac{1}{n}\right) + \left(1 + \frac{1}{n}\right)\frac{1}{n}$$

$$= \left(1 + \frac{1}{n}\right)^2$$



y3 =

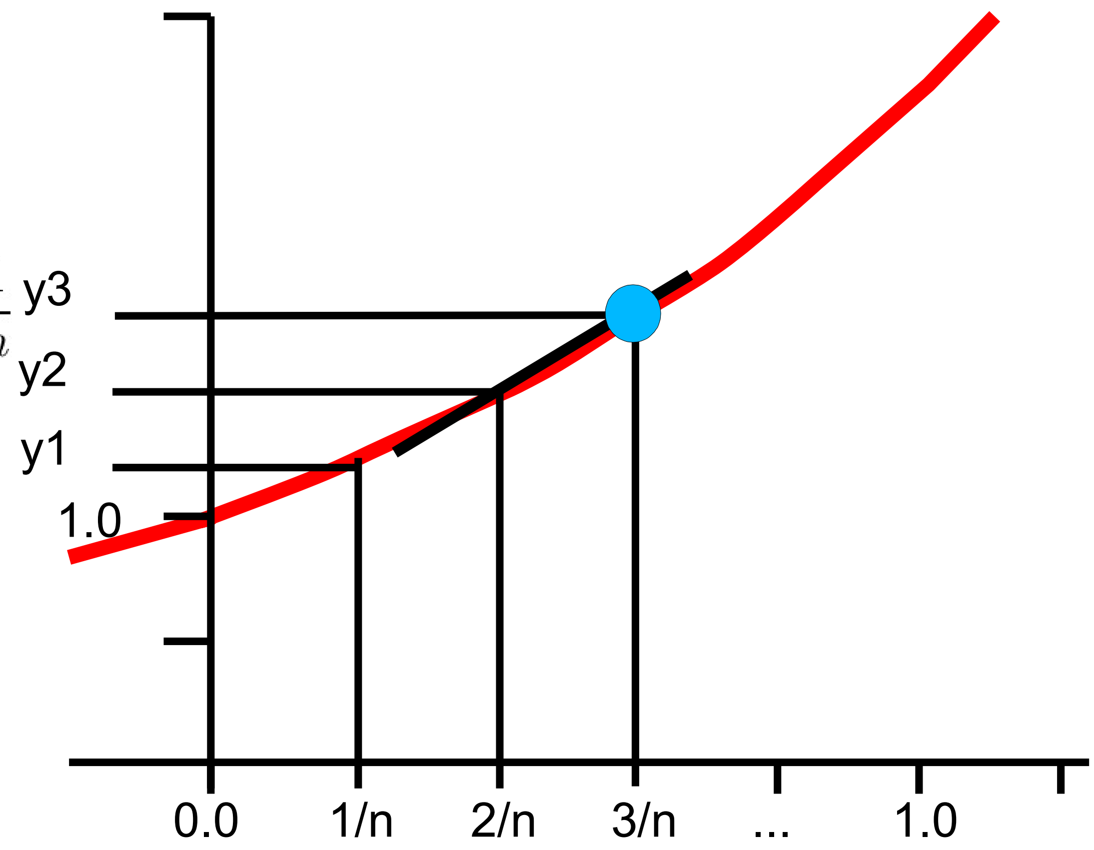
- Taylor expansion again

$$= y_2 + \left(1+\frac{1}{n}\right)^2 \frac{1}{n}$$

$$= \left(1+\frac{1}{n}\right)^2 + \left(1+\frac{1}{n}\right)^2 \frac{1}{n}$$

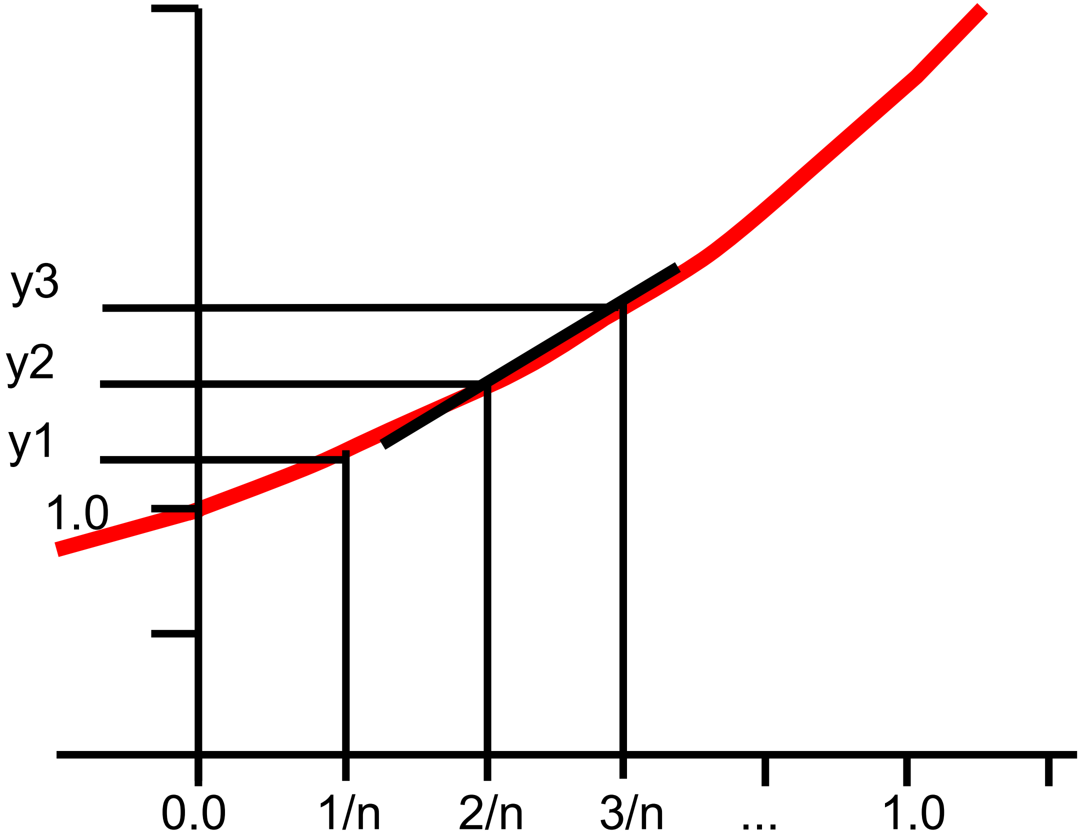
$$= \left(1+\frac{1}{n}\right)^2 \left(1+\frac{1}{n}\right)$$

$$= \left(1+\frac{1}{n}\right)^3$$



Finally: yn

$$y_n = \left(1 + \frac{1}{n}\right)^n$$



Conclusion

- Now we know the definition of *e*.
- Only two assumptions are necessary.
  - Any number powered by 0 is 1
  - Derivative of e^x's is e^x.

$$y_n = \left(1 + \frac{1}{n}\right)^n$$

$$e \equiv \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$$